5.1 Exponential Functions p. 238

x^2 – power function

2^x – exponential

Ex. $u_0 = \$14{,}000$

$u_n = (1 - .20)u_{n-1}$

where $n \geq 1$

$u_n = \cancel{0.8u_{n-1} - 14{,}000}$

$\cancel{14000u_{n-1} - 0.8}$

$\cancel{= 14000 - 0.8n}$

$u_1 = 0.8(14000)$

$u_n = (0.8)^n(14000)$

$u_2 = 0.8(u_1) = 0.8 \cdot 0.8 \cdot 14000$

$= (0.8)^2(14000)$

$u_3 = (0.8)^3(14000)$

$u_n = r^n u_0 = u_0 r^n$

commutative

Exponential Function

$y = a\,b^x$

ratio

p. 240
#1–4

Power Function

$y = a x^b$

^ raised to the power of

$r - 1 =$ − decrease
+ increase

5.2 Properties of Exponents and Power Functions p. 245 2/10/12

Inv. a) $2^3 \cdot 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7$

$b^3 \cdot b^4 = b^7$

b) $x^5 \cdot x^{12} = x^{17}$

c) $10^2 \cdot 10^5 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^7$

$$\boxed{a^m \cdot a^n = a^{m+n}}$$

$$\frac{4^5}{4^2} = \frac{4\cdot 4\cdot \cancel{4}\cdot \cancel{4}\cdot 4}{\cancel{4}\cdot \cancel{4}} = 4^3$$

$$\boxed{\frac{a^m}{a^n} = a^{m-n}}$$

$$\frac{x^8}{x^6} = x^2$$

$$\boxed{\frac{1}{a^m} = a^{-m}}$$

$$\frac{(0.94)^{15}}{(0.94)^5} = (0.94)^{10}$$

$$\frac{2^3}{2^4} = 2^{-1} \qquad \frac{\cancel{2}\cdot \cancel{2}\cdot \cancel{2}}{\cancel{2}\cdot \cancel{2}\cdot \cancel{2}\cdot 2} = \frac{1}{2}$$

$$\frac{4^5}{4^7} = \frac{\cancelto{1}{4}\cdot \cancelto{1}{4}\cdot \cancelto{1}{4}\cdot \cancelto{1}{4}\cdot \cancelto{1}{4}}{\cancel{4}\cdot \cancel{4}\cdot \cancel{4}\cdot \cancel{4}\cdot \cancel{4}\cdot 4\cdot 4} = \frac{1}{4^2}$$

$$= 4^{-2}$$

$(x^4)^2 = (x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x \cdot x) = x^8$

$$\boxed{(a^m)^n = a^{m \cdot n}}$$

$(2^3)^3 = 2^9 \quad 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^9$

$(2^2)^{-2} = 2^{-4}$

$\dfrac{1}{2 \cdot 2 \cdot 2 \cdot 2} = \dfrac{1}{2^4}$

$(3^{-4})^{-7} = 3^{28}$

$\left(\dfrac{1}{3^4}\right)^{-7} = \dfrac{1}{\left(\dfrac{1}{3^4}\right)^7} = (3^4)^7$

Product Property

$$a^m \cdot a^n = a^{m+n}$$

Quotient Property

$$\frac{a^m}{a^n} = a^{m-n}$$

Def. of neg. exponents

$$a^{-n} = \frac{1}{a^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

Zero Exponents

$$a^0 = 1$$

Power of a Power

$$(a^m)^n = a^{m \cdot n}$$

Power of a Product

$$(ab)^m = a^m b^m$$

Power of a Quotient

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Power Property of Equality

If $a=b$ then $a^m = b^m$

Common Base Property of Equality

If $a^m = a^n$ then $m=n$

Ex. A

$8^x = 4$

$(2^3)^x = 2^2$

PoaP

$2^{3x} = 2^2$

CBPE

$\frac{3x}{3} = \frac{2}{3}$

$x = \frac{2}{3}$

$8^{\frac{2}{3}} = 4$

$(2^3)^{\frac{2}{3}} = 2^2$

$27^x = \frac{1}{81}$

$(3^3)^x = \frac{1}{3^4}$

PoaP

$3^{3x} = \frac{1}{3^4}$

Def Neg Exp

$3^{3x} = 3^{-4}$

CBPE

$\frac{3x}{3} = \frac{-4}{3}$

$x = \frac{-4}{3}$

$(3^3)^{-\frac{4}{3}} = \frac{1}{3^4}$

$3^{-4} = \frac{1}{3^4}$

$$\left(\frac{49}{9}\right)^x = \left(\frac{3}{7}\right)^{\frac{3}{2}}$$

$$\left(\frac{7^2}{3^2}\right)^x = \left(\frac{3}{7}\right)^{\frac{3}{2}}$$

PoaQ

$$\left(\frac{7}{3}\right)^{2\cdot x} = \left(\frac{3}{7}\right)^{\frac{3}{2}}$$

DONE

$$\left(\frac{7}{3}\right)^{2x} = \left(\frac{7}{3}\right)^{-\frac{3}{2}}$$

CBPE

$$\frac{2x}{2} = \frac{-\frac{3}{2}}{2}$$

$$x = -\frac{3}{2}\cdot\frac{1}{2} = -\frac{3}{4}$$

Ex 3 a) $x^4 = 3{,}000$ 2/13/12

$$\left(x^4\right)^{\frac{1}{4}} = 3000^{\frac{1}{4}}$$

3000^(1/4)

$$x^1 = 3000 \wedge .25$$

$$x \simeq 7.40$$

$$\frac{6x^3}{6} = \frac{2400}{6}$$

$$\left(x^3\right)^{\frac{1}{3}} = 400^{\frac{1}{3}}$$

$$x^1 \simeq 7.37$$

b) $\frac{6x^{2.5}}{6} = \frac{90}{6}$

$x^{2.5} = 15$

$\left(x^{\frac{5}{2}}\right)^{\frac{2}{5}} = 15^{\frac{2}{5}}$

$x^1 \simeq 2.95$

Power function

$y = ax^n$

Exponential function

$y = ab^x$

p.248

#1-41

1a) $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

Done

$a \cdot b^x = (ab)^x$

rational - pertaining to a ratio

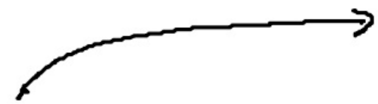
ratio - comparison of two numbers $5:2$, $\frac{5}{2}$

Inv. $y_1 = X \wedge (1/2)$

$\sqrt{y_1^2} = \sqrt{x}$

$y_1 = \sqrt{x} = x^{\frac{1}{2}}$

ellipse



power

$y_1 = 25^x$

exponential

$49^{\frac{3}{2}} = 343$

$(7^2)^{\frac{3}{2}} = 7^{(2\cdot\frac{3}{2})} = 7^3 = 343$

$(a^m)^n = a^{m\cdot n}$

$27^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^2 = 9$

$27^{\frac{2}{3}} = (27^2)^{\frac{1}{3}} \quad \sqrt[3]{27^2}$

$$\boxed{\begin{aligned} a^{\frac{m}{n}} &= (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m} \\ a^{\frac{m}{n}} &= (a^{\frac{1}{n}})^m = (\sqrt[n]{a})^m \end{aligned}}$$

Definition of Rational Exponents

Ex. A

a) $\sqrt[4]{a} = 14$

$(a^{\frac{1}{4}})^4 = 14^4$

$a = 38{,}416$

b) $\sqrt[9]{b^5} = 26$

$(b^5)^{\frac{1}{9}} = 26$

$(b^{\frac{5}{9}})^{\frac{9}{5}} = (26)^{\frac{9}{5}}$

$b = 352.33$

c) $(\sqrt[3]{c})^8 = 47$ $\qquad (\sqrt[3]{c})^8 = \sqrt[3]{c^8}$

$(c^{\frac{8}{3}})^{\frac{3}{8}} = 47^{\frac{3}{8}}$

$c \approx 4.237$

$(a^m)^{\frac{1}{n}}$ if m is even, a can be <0

point-slope form of a line

$y = y_1 + b(x - x_1)$

$$\boxed{\begin{array}{l}\text{point-ratio form for exponentials} \\ y = y_1 \cdot b^{(x - x_1)}\end{array}}$$

Ex B $(7s, 4.7\frac{lb}{in^2})$

$(4s, 40\frac{lb}{in^2})$

2/22/12

$$y = y_1 \cdot b^{(x-x_1)}$$

$$y = 40 \cdot b^{(x-4)}$$

$$y = 4.7 \cdot b^{(x-7)}$$

$$\frac{40 \cdot b^{(x-4)}}{40} = \frac{4.7 \cdot b^{(x-7)}}{40}$$

$$\frac{b^{(x-4)}}{b^{(x-7)}} = \frac{4.7}{40} \frac{b^{(x-7)}}{b^{(x-7)}}$$

$$\frac{b^{(x-4)}}{b^{(x-7)}} = \frac{4.7}{40}$$

$$b^{(x-4)-(x-7)} = .1175$$

$$\sqrt[3]{b^3} = \sqrt[3]{.1175}$$

$$b \approx .4898$$

$$y = 40 \cdot 0.4898^{(x-4)}$$

$$y = 40 \cdot 0.4898^{(0-4)}$$

$$y = 695 \frac{lb}{in^2} \text{ at } t = 0s$$

pp. 256-259
#1-4, 6

5.4 Applications of Exponential and Power Equations p. 261 2/24/12

Ex. A $500 8yr $1000 comp. annually

$u_n = u_0 r^n$

$\frac{1000}{500} = \frac{500}{500}(1+r)^8$

$2 = 2^1$ $2^{1/8} = (1+r)^{8/8}$

$2^{1/8} = 1 + r$
$-1 \qquad -1$

$2^{1/8} - 1 = r$

$.09 = r$

9%

$\frac{4000}{2000} = \frac{2000}{2000}(1+r)^8$

$2 = (1+r)^8$

$4 = (1+r)^8$

$r = .19$

Ex. B point-ratio form

$y = y_1 b^{(x-x_1)}$

$y = 2.35 b^{(x-0)}$

$y = 1.97 b^{(x-10)}$

$\frac{2.35}{1.97} b^x = \frac{1.97}{1.97} b^{(x-10)}$

$1.19 \frac{b^x}{b^x} = \frac{b^{(x-10)}}{b^x}$

$1.19 = b^{(x-10)-x}$

$1.19^{-\frac{1}{10}} = b^{-10 \cdot -\frac{1}{10}}$

$b = 1.19^{-\frac{1}{10}} = .983$

$y = 2.35 \cdot .983^x$

p. 263
#1–5

1) $(x^5)^{\frac{1}{5}} = x^1$ $\quad 50^{\frac{1}{5}} = x$

b) $\sqrt[3]{x} = x^{\frac{1}{3} \cdot 3}$

$3.1^{1 \cdot 3}$ $\quad x^1 = 3.1^3$

2) a) $x^{\frac{1}{4}} - 2 = 3$

$\quad +2 \qquad +2$

$x^{\frac{1}{4} \cdot 4} = 5^{\cdot 4}$

$X = 5^{4}$

$x = 25^{2}$

$x = 625$

PEMDAS

3) $(27x^{6})^{\frac{2}{3}}$

$(ab)^{m} = a^{m} b^{m}$

$27^{\frac{2}{3}}(x^{6})^{\frac{2}{3}} = (3^{3})^{\frac{2}{3}}(x^{6})^{\frac{2}{3}}$

$3^{2}x^{4} = 9x^{4}$

b) $(16x^{8})^{\frac{3}{4}} = 16^{3/4}(x^{8})^{\frac{3}{4}} = (2^{4})^{\frac{3}{4}}(x^{8})^{\frac{3}{4}}$

$2^{3}x^{6} = 8x^{6}$

2d) $\div 800$

$n^{1/7.8}$

-1

$*12$

$X = .951$

4) 6 sheets → 50% reduction

a^m $\qquad 50^{1/6} = (1-r)^{6/6}$

$50^{1/6} = 1 - r$

$-1 \qquad -1$

$-r = 50^{\frac{1}{6}} - 1$

$x-1 \qquad x-1$

$r = 1 - 50^{\frac{1}{6}}$

$r = -.919$

8.1%

5.5 Building Inverses of Functions p. 266 2/29/12

inverse - switch the dependent and independent variables

Inv.

i. $f(x) = 6 + 3x$

ii. $f(x) = \sqrt{(x+4)} - 3$

iii. $f(x) = (x-2)^2 - 5$

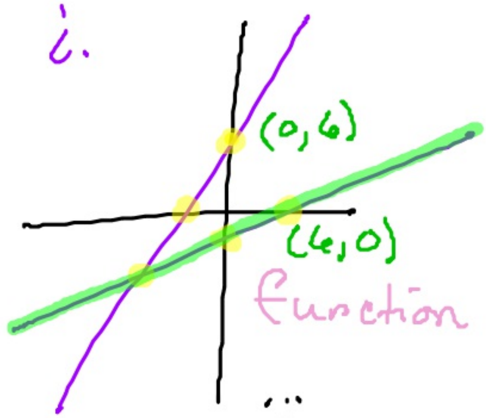
iv. $f(x) = 2 + \sqrt{x+5}$

v. $f(x) = \frac{1}{3}(x-6)$

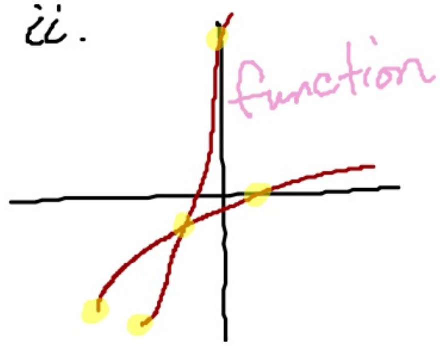
vi. $f(x) = \sqrt[3]{5x}$

Sketch + write down 3 points from original + inverse

i.

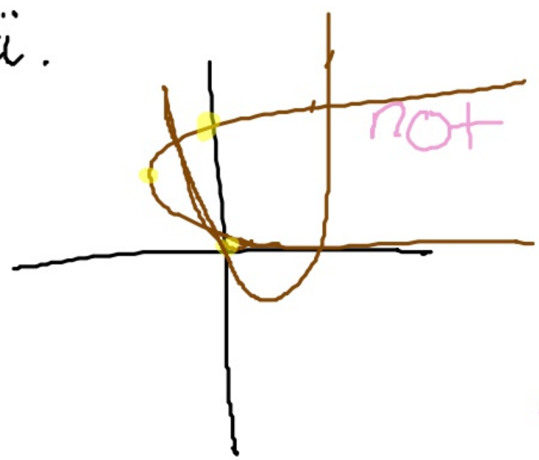


ii.

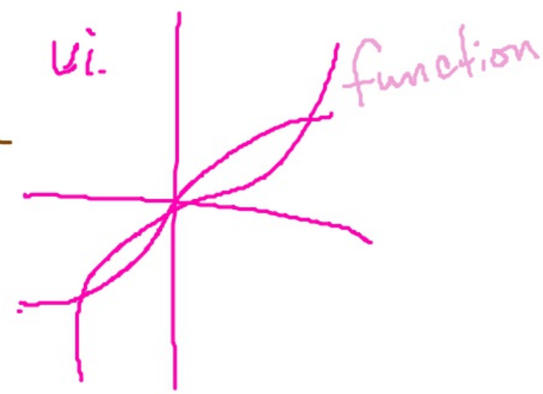


	x y	x y
i.	(0,6)	(6,0)
	(-2,0)	(0,-2)
	(-3,-3)	(-3,-3)

iii.



vi.



Find the inverse of a function by exchanging the independent (x) variable and the dependent variable (y).

$y = 6x + 2$

$x = 6y + 2$
$-2 \qquad\quad -2$

$\frac{x-2}{6} = \frac{6y}{6}$

$y = \frac{1}{6}(x-2)$

$y = \frac{1}{6}(x-2)$

p. 269–270
#1–4

$y = x^2 + 2$

$x = y^2 + 2$
$-2 \qquad\quad -2$

$\sqrt{x-2} = \sqrt{y^2}$

$y = \sqrt{x-2} = (x-2)^{\frac{1}{2}}$

Ex. B $f(x) = 4 - 3x$

$y = 4 - 3x$

$x = 4 - 3y$

$-4 \quad -4$

$\frac{x-4}{-3} = \frac{-3y}{-3}$

$y = -\frac{1}{3}(x-4)$

$f^{-1}(x) = -\frac{1}{3}(x-4)$

$y = ab^x$

$y = a(1+r)^x$

$-r$

$f^{-1}(9) = -\frac{1}{3}(x-4)$

9

$9 - 4 = \frac{5}{1}$

$f^{-1}(9) = -\frac{5}{3}$

$\frac{5}{1} \times \frac{-1}{3} = \frac{-5}{3}$

P arentheses

E xponents $\left(\frac{1}{2}\right)^{1/8}$

M ultiply

D ivide

A dd

S ubtract

5.6 Logarithmic Functions p. 273 3/12/12

$10^x = 1000$ $3^x = 81$ factoring

$x = 3$ $x = 4$

$4^x = \frac{1}{16}$ $\frac{1}{4} \cdot \frac{1}{4}$ $\frac{1}{4^2}$ DONE 4^{-2}

Logarithm $\log_{10}(\ ) = \text{Log}(\ )$

common logarithm

10^x ~~Ln~~

Inv. $Y_1 = 10^x$ Compare Y_1 and Y_2 values in a table. 3/14/12

$Y_2 = \log(10^x)$ What do you notice?

$x=2$ $Y_1 = 10^2 = 100$ $Y_2 = \log(10^2) = 2$

$\log(10^x) = x$

$\log(10^{2.5}) = 2.5$

$\log(10^{-3.2}) = -3.2$

$\log(10^0) = 0$

$\log(10^x) = x$

Ex. A

Solve $\frac{4 * 10^x}{4} = \frac{4650}{4}$

$$10^x = 1162.5$$

$$\log(10^x) = \log(1162.5)$$

$$x = 3.06\underline{5}$$

Ex. B

Solve $4^x = 128$

10^x $\quad 10^b = 4$

$\log(10^b) = \log(4)$

$b = 0.602\ldots$

Coefficient

Change of base

$$\log_b(a) = \frac{\log(a)}{\log(b)} \qquad a>0 \quad b>0$$

$(10^{0.6021})^x = 128$

$\log(10^{0.6021x}) = \log(128)$

$0.6021x = \log(128)$

$$x = \frac{\log(128)}{\log(4)}$$

$x \approx 3.5$

Ex. C \$500 $\longrightarrow$ \$800 12% annually

$$y = a(1+r)^x$$

$$800 = 500(1+.12)^x$$

$$\frac{800}{500} = \frac{500}{500}(1.12)^x$$

$$1\tfrac{3}{5} = (1.12)^x$$

$$\log(1.6) = \log(1.12^x)$$

$$\frac{\log(1.6)}{\log(1.12)} = x \approx 4.15$$

Def. of log

$\log_b(a) = x$ $\qquad a = b^x$

$a > 0,\ b > 0$

p. 276
#1–4

$\frac{1}{25} = \frac{1}{5^2} = 5^{-2}$

#3 #1 $x = \log_{10}(0.001)$

$10^x = 0.001$

$6^x = 1$

DOZE $a^0 = 1$

$\log_5(100) = x$

base

$\log_{0.8} 0.03 = x$

$\log_8 35$ $\log_{35} 8$

$\log_{17} 0.5 = x$

Ex. A (a) Multiply 183.47 by 19.628 without using the "X" key on the calculator.

$(183.47)(19.628) \approx 10^{3.5566} \approx 3602.4669$ 3/19/12

$\log(183.47) = 10^x = 183.47 \quad x = 2.2637$

$\log(19.628) = 10^x = 19.628 \quad x = 1.2929$

$10^{2.2637} \cdot 10^{1.2929} = 10^{2.2637 + 1.2929} = 10^{3.5566}$

b) Divide 183.47 by 19.628 without using the " ÷ " key on the calculator.

10^x $\quad 10^{2.2637} / 10^{1.2929} = 10^{2.2637-1.2929}$

$\approx 10^{0.9708} \approx 9.3498$

c) Evaluate $4.70^{2.8}$ without using the " ^ " key on your calculator.

$4.70^{2.8} = X$

$\log(4.70) \approx 10^{0.6721}$

$\left(10^{0.6721}\right)^{2.8} = x = 10^{0.6721 \times 2.8}$

$X = 10^{1.8819} \approx 76.1904$

2nd log $= 10^{x}$

Properties of Exponents and Logarithms

3/21/12

Definition of a logarithm

If $x = a^m$, then $\text{Log}_a x = m$.

Product Property

$$a^m \cdot a^n = a^{m+n}$$

$$\log_b xy = \log_b x + \log_b y$$

Quotient Property

$$\frac{a^m}{a^n} = a^{m-n} \qquad \log_b \frac{x}{y} = \log_b x - \log_b y$$

Power Property

$$\text{Log}_a X^n = n \log_a X$$

Power of a Power Property

$$(X^a)^b = X^{a \cdot b}$$

Power of a Product Property

$$(ab)^m = a^m b^m$$

Power of a Quotient Property

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Change of Base Property

$$\text{Log}_a x = \frac{\text{Log}_b x}{\text{Log}_b a}$$

Definition of Rational Exponents

$$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$$

Definition of Negative Exponents (DONE)

$$a^{-n} = \frac{1}{a^n} \text{ or } \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

Definition of Zero Exponents (DOZE)

$$a^0 = 1$$

p. 282 - 284
1, 2, 5

Ex. A. pendulum from 5.4, closest to 1.47m

$$y = 1.25 + 0.72(0.954)^{x-10}$$

$$1.47 = 1.25 + 0.72(0.954)^{x-10}$$

$$-1.25 \quad -1.25$$

$$\frac{0.22}{0.72} = \frac{0.72(0.954)^{x-10}}{0.72}$$

$$0.306 = 0.954^{x-10}$$

$$\frac{\log(0.306)}{\log(0.954} = \frac{(x-10)\log(0.954)}{\log(0.954)}$$

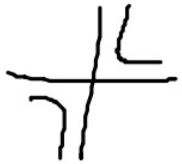
$$\frac{\log(0.306)}{\log(0.954)} = x - 10$$

$$+10 \quad +10$$

$$\frac{\log\left(\frac{0.22}{0.72}\right)}{\log(0.954)} + 10 = x$$

$$\log(0.22) - \log(0.72) - \log(0.954) + 10 = x$$

Ex. B



$y = k + ab^x$

$\log(x) = \log(k + ab^x)$

$c \qquad \log(ab^x)$

$\log(a) + \log(b^x)$

$\log(y) = \log(a) + x\log(b)$

$\log(y) = c + d\,x$

pp. 285-290

#1-41